Copying and permutation in substructural logics

McPheat, Wazni, Sadrzadeh: UCL

Pregroup Grammars *Lambek 1999*

$$(P, \cdot, 1, \leq, (-)^r, (-)^l)$$

Partially Ordered Monoid

$$a, b, c \in P \qquad a \leq b \implies a \cdot c \leq b \cdot c \quad \text{and} \quad c \cdot a \leq c \cdot b$$

Left and Right Pre-Inverses

$$\forall a \in P, \exists a^r, a^l \in P \qquad a \cdot a^r \leq 1 \leq a^r \cdot a \quad \text{and} \quad a^l \cdot a \leq 1 \leq a \cdot a^l$$

Some Features

Antitonicity $$a \leq b \implies b^l \leq a^l$$

$$(a \cdot b)^l \leq b^l \cdot a^l$$

Pre-Inverses are not idempotent $$a^{ll} \neq a \neq a^{rr}$$

Iterative Pre-Inverses $$a^{lll} \cdot a^{ll} \leq 1 \leq a^{rr} \cdot a^{rrr}$$

$$a^{ll} \cdot a^l \leq 1 \leq a^r \cdot a^{rr}$$

Cancelling Pre-Inverses $$a^{lr} = a^{rl} = a$$

Applications to Natural Language

(I) Fix a set of basic types ***B*** and a partial ordering on it.

(II) Assign a set of elements from the (free) pregroup generated over ***B*** to each element of the vocabulary of our natural language.

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$$n, s, q$$

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Applications to Natural Language

(I) Fix a set of basic types and a partial ordering on it ***B***.

$$n, s, q$$

$$n_0, n_1, n_2 \qquad s_1, s_2, i \qquad \bar{q}, q_1, q_2$$

$$s_i \leq s \qquad q_i \leq q \qquad n_i \leq n$$

(II) Assign a set of types from the pregroup generated over ***B*** to each element of the vocabulary.

Simple Examples

Men sleep. $n \cdot (n^r \cdot s) \quad \leq \quad s$

Mary slept.

Mary saw John.

Mary may see John.

Simple Examples

Men sleep.	$n_2 \cdot (n_2^r \cdot s_1)$	$\leq$	s_1
Mary slept.	$n_0 \cdot (n_0^r \cdot s_2)$	$\leq$	s_2
Mary saw John.	$n_0 \cdot (n_0^r \cdot s_2 \cdot n_0^l) \cdot n_0$	$\leq$	s_2
Mary may see John.	$n_0 \cdot (n_0^r \cdot s_1 \cdot i^l) \cdot (i \cdot n_0^l) \cdot n_0$	$\leq$	s_1

Elaborate Examples

Some men sleep.

$$(n \cdot n^l) \cdot n \cdot (n^r \cdot s_1) \qquad \leq \qquad s_1$$

All men die.

Some people eat raw meat.

$$(n \cdot n^l) \cdot n \cdot (n^r \cdot s \cdot n^l) \cdot (n \cdot n^l) \cdot n \qquad \leq \qquad s$$

More Elaborate Examples

Some tall men and old women slept.

$$(n \cdot n^l) \cdot (n \cdot n^l) \cdot n \cdot (n^r \cdot n \cdot n^l) \cdot (n \cdot n^l) \cdot n \cdot (n^r \cdot s) \qquad \leq \qquad s$$

All men live and die.

$$(n \cdot n^l) \cdot n \cdot (n^r \cdot s) \cdot ((n^r \cdot s)^r \cdot (n^r \cdot s) \cdot (n^r \cdot s)^l) \cdot (n^r \cdot s) \qquad \leq \qquad s$$

Who eats vegetables?

$$(\bar{q} \cdot s^l \cdot n) \cdot (n^r \cdot s \cdot n^l) \cdot n \qquad \leq \qquad \bar{q}$$

People who eat vegetables.

$$n \cdot (n^r \cdot n \cdot (n^r \cdot s)^l) \cdot (n^r \cdot s \cdot n^l) \cdot n \qquad \leq \qquad n$$

Movement in Hungarian

Basic word order:

János	tegnap	elvitt	két könyvet	Péternek.
János	yesterday	took	two books	to Péternek.

Movement for mild emphasis:

János	tegnap	**két könyvet**	vitt el	Péternek.
János	tegnap	**Péternek**	vitt el	két könyvet.

Movement as a result of more information:

Péternek	vitt el	tegnap	János	két könyvet.
Két könyvet	vitt el	János	tegnap	Péternek.

Movement in Sanskrit

Basic word order:

Ramah	*apasyat*	*Govindam.*
Rama	*saw*	*Govinda*

Other orders:

Ramah Govindam apasyat.

apasyat Govindam Ramah.

Govindam apasyat Ramah.

Wrong orders:

$(*_1)$ *Govindam Ramah apasyat* $(*_2)$ *apasyat Ramah Govindam*

Movement in Persian

Basic word order:

Hassan   Nadia   saw.

Hassan Nadia-ra Did.

Other possible orders:

Nadia-ra Hassan Did.

did Nadia-ra Hassan.

did Hassan Nadia-ra.

Movement in Persian

Basic word order:

Hassan Nadia-ra did.

Nadia-ra Hassan did.

n n $n^r \cdot n^r \cdot s$

Other possible orders:

did Nadia-ra Hassan.

did Hassan Nadia-ra.

$s \cdot n^l \cdot n^l$ n n

Meta Rules for Movement

Left to right movement:

If a word has type $a \cdot b^l$ then it also has type $b^r \cdot a$.

Right to left movement:

If a word has type $b^r \cdot a$ then it also has type $a \cdot b^l$.

Left to Right Movement

If a word has type $a \cdot b^l$ then it also has type $b^r \cdot a$.

Hassan Nadia saw.

Hassan Nadia-ra did.

Nadia-ra Hassan did.

n n $n^r \cdot n^r \cdot s$

The word `did' has type $(n \cdot n)^r \cdot s$ and also type $s \cdot (n \cdot n)^l$.

did Nadia-ra Hassan.

did Hassan Nadia-ra.

$s \cdot n^l \cdot n^l$ n n

Right to Left Movement

If a word has type $b^r \cdot a$ then it also has type $a \cdot b^l$.

Ramah saw Govindam.

Ramah apasyat Govindam.

$n \quad n^r \cdot s \cdot n^l \quad n$

`apasyat' has type $n^r \cdot s \cdot n^l$ and also type $s \cdot n^l \cdot n^l$.

apasyat Govindam Ramah.

$s \cdot n^l \cdot n^l \quad n \quad n$

Right to Left Movement

If a word has type $b^r \cdot a$ then it also has type $a \cdot b^l$.

Ramah saw Govindam.

Ramah apas[illegible]dam.

n [illegible] n^l n

`apasyat' h[illegible] $n^{[illegible]} \cdot s \cdot n^l$ and also type $s \cdot n^l \cdot n^l$.

Can very easily produce all possible word orders.

apasyat Govindam Ramah.

$s \cdot n^l \cdot n^l$ n n

Meta Rules for Movement

Left to right movement:

the original type of

If a ~~word~~ has type $a \cdot b^l$ then it also has type $b^r \cdot a$.

Right to left movement:

the original type of

If a ~~word~~ has type $b^r \cdot a$ then it also has type $a \cdot b^l$.

Why not axioms?

Definition. A *left cyclic pregroup* is a pregroup in which moreover the following holds:

$$a \cdot b^l \quad \leq \quad b^r \cdot a$$

Definition. A *right cyclic pregroup* is a pregroup in which moreover the following holds:

$$b^r \cdot a \quad \leq \quad a \cdot b^l$$

Proposition. Any of the cyclic pregroups is a partially ordered group.

Beyond Sentence

John Slept. He snored.

$$n \cdot (n^r \cdot s) \cdot (n^l \cdot n) \cdot (n^r \cdot s) \leq s \cdot s$$

Copying

$$n \cdot n \cdot (n^r \cdot s) \cdot (n^l \cdot n) \cdot (n^r \cdot s) \leq s \cdot s$$

Permutation

$$n \cdot (n^r \cdot s) \cdot n \cdot (n^l \cdot n) \cdot (n^r \cdot s) \leq s \cdot s$$

Cyclic Pregroups

Notational conveniences

$$\widetilde{a} \qquad a^* \qquad a'$$

Meta rules

(I) If a word has type $\widetilde{a}$ *, it also has type* $a^* \cdot \cdots \cdot a^*$ *for a fixed number of copies of a.*

(II) If the concatenation of two words ends with type $a^* \cdot b$ *it also ends with type* $b \cdot a^*$

(III) Any word that has type a^* *also has type* a

Translation to Residuated Monoids

A residuated monoid is a partially ordered monoid

$$(L, \cdot, 1, \leq, /, \backslash)$$

in which we have

$$a \cdot (a \backslash b) \leq b \qquad (a/b \cdot b) \leq a$$

If . is commutative, you get something like a Heyting Algebra.

$$a \wedge (a \rightarrow b) \leq b$$

Translation to Residuated Monoids

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$$(L, \cdot, 1, \leq, /, \backslash)$$

in which we have

$$a \cdot a \backslash b \leq b \qquad a/b \cdot b \leq a$$

A pregroup can be translated to a residuated monoid via the following mapping:

$$a \cdot b^l \rightsquigarrow a/b \qquad a^r \cdot b \rightsquigarrow a \backslash b$$

Lambek Calculus

$$\frac{}{A \longrightarrow A}\ I$$

$$\frac{\Gamma \longrightarrow A \quad \Sigma_1, B, \Sigma_2 \longrightarrow C}{\Sigma_1, \Gamma, A\backslash B, \Sigma_2 \longrightarrow C}\ \backslash_L \qquad \frac{A, \Gamma \longrightarrow B}{\Gamma \longrightarrow A\backslash B}\ \backslash_R$$

$$\frac{\Gamma \longrightarrow A \quad \Sigma_1, B, \Sigma_2 \longrightarrow C}{\Sigma_1, B/A, \Gamma, \Sigma_2 \longrightarrow C}\ /_L \qquad \frac{\Gamma, A \longrightarrow B}{\Gamma \longrightarrow B/A}\ /_R$$

$$\frac{\Gamma_1, A, B, \Gamma_2 \longrightarrow C}{\Gamma_1, A \cdot B, \Gamma_2 \longrightarrow C}\ \cdot_L \qquad \frac{\Gamma_1 \longrightarrow A \quad \Gamma_2 \longrightarrow B}{\Gamma_1, \Gamma_2 \longrightarrow A \cdot B}\ \cdot_R$$

Lambek (Am. Maths. Monthly1958)

Lambek Calculus

$$\frac{}{A \longrightarrow A}\ I$$

$$\frac{\Gamma \longrightarrow A \quad \Sigma_1, B, \Sigma_2 \longrightarrow C}{\Sigma_1, \Gamma, A\backslash B, \Sigma \ldots}$$

$$\frac{\ldots \longrightarrow B}{\Gamma \longrightarrow A\backslash B}\ \backslash_R$$

$$\frac{\Gamma \ldots \Sigma_2 \longrightarrow C}{\ldots A, \Gamma, \Sigma_2 \longrightarrow C}\ /_L$$

$$\frac{\Gamma, A \longrightarrow B}{\Gamma \longrightarrow B/A}\ /_R$$

$$\frac{\Gamma_1, A, B, \Gamma_2 \longrightarrow C}{\Gamma_1, A \cdot B, \Gamma_2 \longrightarrow C}\ \cdot_L$$

$$\frac{\Gamma_1 \longrightarrow A \quad \Gamma_2 \longrightarrow B}{\Gamma_1, \Gamma_2 \longrightarrow A \cdot B}\ \cdot_R$$

Unintuitive Derivations, but has a decision procedure.

Adding copying and permutation

$$\frac{\Gamma_1, A, \Gamma_2 \to C}{\Gamma_1, !A, \Gamma_2 \to C}\ (!L) \qquad \frac{!A_1, \ldots, !A_n \to B}{!A_1, \ldots, !A_n \to !B}\ (!R)$$

$$\frac{\Delta_1, !A, \Gamma, \Delta_2 \to C}{\Delta_1, \Gamma, !A, \Delta_2 \to C}\ (perm_1) \qquad \frac{\Delta_1, \Gamma, !A, \Delta_2 \to C}{\Delta_1, !A, \Gamma, \Delta_2 \to C}\ (perm_2)$$

$$\frac{\Delta_1, !A, !A, \Delta_2 \to C}{\Delta_1, !A, \Delta_2 \to C}\ (contr)$$

Kanovich, Kuznetsov, Scedrov (FG 2017)

Adding copying and permutation

$$\frac{\Gamma_1, A, \Gamma_2 \longrightarrow C}{\Gamma_1, !A, \Gamma_2 \longrightarrow C}\ (!L)$$

$$\frac{!A_1, \ldots, !A_n \longrightarrow B}{!A_1 \ldots}\ (!R)$$

$$\frac{\Delta_1, !A, \Gamma, \Delta_2 \longrightarrow \ldots}{\Delta_1, \Gamma, ! \ldots}\ (\ldots 1)$$

$$\frac{\Delta_1, \Gamma, !A, \Delta_2 \longrightarrow C}{\Delta_1, !A, \Gamma, \Delta_2 \longrightarrow C}\ (perm_2)$$

$$\frac{\Delta_1, !A, !A, \Delta_2 \longrightarrow C}{\Delta_1, !A, \Delta_2 \longrightarrow C}\ (contr)$$

Overgenerates hugely and is undecidable!

Separating copying from permutation

$$\frac{\Gamma_1, \overbrace{A, A, \ldots, A}^{n \text{ times}}, \Gamma_2 \longrightarrow B}{\Gamma_1, !A, \Gamma_2 \longrightarrow B}\ !_L \qquad \frac{A \longrightarrow B}{!A \longrightarrow !B}\ !_R$$

$$\frac{\Gamma_1, A, \Gamma_2 \longrightarrow B}{\Gamma_1, \nabla A, \Gamma_2 \longrightarrow B}\ \nabla_L \qquad \frac{A \longrightarrow B}{\nabla A \longrightarrow \nabla B}\ \nabla_R$$

$$\frac{\Gamma_1, \Gamma_2, \nabla A, \Gamma_3 \longrightarrow B}{\Gamma_1, \nabla A, \Gamma_2, \Gamma_3 \longrightarrow B}\ perm \qquad \frac{\Gamma_1, \nabla A, \Gamma_2, \Gamma_3 \longrightarrow B}{\Gamma_1, \Gamma_2, \nabla A, \Gamma_3 \longrightarrow B}\ perm'$$

Kanovich, Kuznetsov, Nigam, Scedrov (IJCAR 2020)

Separating copying from permutation (bounding copies)

$$\frac{\Gamma_1, \overbrace{A, A, \ldots, A}^{n \text{ times}}, \Gamma_2 \longrightarrow B}{\Gamma_1, !A, \Gamma_2 \longrightarrow B}\ !_L$$

A — $!_R$

Γ_1, A, Γ … B ∇_L

Overgenerates less and is decidable!

$$\frac{A \longrightarrow B}{\nabla A \longrightarrow \nabla B}\ \nabla_R$$

$$\frac{\Gamma_1, \Gamma_2, \nabla A, \Gamma_3 \longrightarrow B}{\Gamma_1, \nabla A, \Gamma_2, \Gamma_3 \longrightarrow B}\ perm \qquad \frac{\Gamma_1, \nabla A, \Gamma_2, \Gamma_3 \longrightarrow B}{\Gamma_1, \Gamma_2, \nabla A, \Gamma_3 \longrightarrow B}\ perm'$$

Example Derivation

$$\dfrac{\overline{n \longrightarrow n} \qquad \overline{s, s \longrightarrow s, s}}{s, n, n\backslash s \longrightarrow s, s}\ \backslash_L$$

$$\dfrac{\overline{\nabla n \longrightarrow \nabla n} \qquad s, n, n\backslash s \longrightarrow s, s}{s,\ \nabla n,\ \nabla n\backslash n, n\backslash s \longrightarrow s, s}\ \backslash_L$$

$$\dfrac{\overline{n \longrightarrow n} \qquad s,\ \nabla n,\ \nabla n\backslash n, n\backslash s \longrightarrow s, s}{n, n\backslash s,\ \nabla n,\ \nabla n\backslash n, n\backslash s \longrightarrow s, s}\ \backslash_L$$

$$\dfrac{\dfrac{\dfrac{n, n\backslash s,\ \nabla n,\ \nabla n\backslash n, n\backslash s \longrightarrow s, s}{\nabla n, n\backslash s,\ \nabla n,\ \nabla n\backslash n, n\backslash s \longrightarrow s, s}\ \nabla_L}{\nabla n,\ \nabla n, n\backslash s,\ \nabla n\backslash n, n\backslash s \longrightarrow s, s}\ perm}{!(\nabla n), n\backslash s,\ \nabla n\backslash n, n\backslash s \longrightarrow s, s}\ contr_2$$

John slept. He snored!

Example Derivation

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{n \longrightarrow n} \qquad \dfrac{\overline{\nabla(n\backslash s) \longrightarrow \nabla(n\backslash s)} \qquad \dfrac{\overline{n \longrightarrow n} \qquad \overline{s, s \longrightarrow s, s}}{s, n, n\backslash s \longrightarrow s, s}\,\backslash_L}{s, n,\ \nabla(n\backslash s),\ \nabla(n\backslash s)\backslash n\backslash s \longrightarrow s, s}\,\backslash_L}{n, n\backslash s, n,\ \nabla(n\backslash s),\ \nabla(n\backslash s)\backslash n\backslash s \longrightarrow s, s}\,\backslash_L}{n,\ \nabla(n\backslash s), n,\ \nabla(n\backslash s),\ \nabla(n\backslash s)\backslash n\backslash s \longrightarrow s, s}\,\nabla_L}{n,\ \nabla(n\backslash s),\ \nabla(n\backslash s), n,\ \nabla(n\backslash s)\backslash n\backslash s \longrightarrow s, s}\,perm}{n, !(\nabla(n\backslash s)), n,\ \nabla(n\backslash s)\backslash n\backslash s \longrightarrow s, s}\,contr_2
$$

Andy plays violin. Ada does too.

Strict vs Sloppy

$$\dfrac{n \longrightarrow n \qquad s, s \longrightarrow s, s}{s, n, n\backslash s \longrightarrow s, s}\ \backslash_L$$

$$\dfrac{n \longrightarrow n \qquad s, n, n\backslash s \longrightarrow s, s}{n, n\backslash s, n, n\backslash s \longrightarrow s, s}\ \backslash_L$$

$$\dfrac{n, n\backslash s, n, n\backslash s \longrightarrow s, s}{n, n\backslash s,\ \nabla n, n\backslash s \longrightarrow s, s}\ \nabla_L$$

$$\dfrac{n, n\backslash s,\ \nabla n, n\backslash s \longrightarrow s, s}{n,\ \nabla(n\backslash s),\ \nabla n, n\backslash s \longrightarrow s, s}\ \nabla_L$$

$$\dfrac{n,\ \nabla(n\backslash s),\ \nabla n, n\backslash s \longrightarrow s, s}{\nabla n,\ \nabla(n\backslash s),\ \nabla n, n\backslash s \longrightarrow s, s}\ \nabla_L$$

$$\dfrac{\nabla n,\ \nabla(n\backslash s),\ \nabla n, n\backslash s \longrightarrow s, s}{\nabla n,\ \nabla(n\backslash s), !(\nabla n), n\backslash s \longrightarrow s, s}\ contr_1$$

$$\dfrac{\nabla(n\backslash s) \longrightarrow\ \nabla(n\backslash s) \qquad \nabla n,\ \nabla(n\backslash s), !(\nabla n), n\backslash s \longrightarrow s, s}{\nabla n,\ \nabla(n\backslash s), !(\nabla n),\ \nabla(n\backslash s),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ \backslash_L$$

$$\dfrac{\nabla n,\ \nabla(n\backslash s), !(\nabla n),\ \nabla(n\backslash s),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}{\nabla n,\ \nabla(n\backslash s),\ \nabla(n\backslash s), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ perm$$

$$\dfrac{\nabla n,\ \nabla(n\backslash s),\ \nabla(n\backslash s), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}{\nabla n, !(\nabla(n\backslash s)), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ contr_2$$

$$\dfrac{n \longrightarrow n \qquad \nabla n, !(\nabla(n\backslash s)), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}{\nabla n, !(\nabla(!(\nabla(n\backslash s))/n)), n, !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ /_L$$

$$\dfrac{\nabla n \longrightarrow\ \nabla n \qquad \nabla n, !(\nabla(!(\nabla(n\backslash s))/n)), n, !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}{\nabla n, !(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\ \nabla n\backslash n, !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ \backslash_L$$

$$\dfrac{\nabla n, !(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\ \nabla n\backslash n, !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}{\nabla n, !(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\ \nabla(\nabla n\backslash n), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ \nabla_L$$

$$\dfrac{\nabla n, !(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\ \nabla(\nabla n\backslash n), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}{\nabla n, !(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n, !(\nabla(\nabla n\backslash n)), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ contr_1$$

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$$\dfrac{\nabla n,\ \nabla n, !(\nabla(!(\nabla(n\backslash s))/n)), !(\nabla(\nabla n\backslash n)), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}{!(\nabla n), !(\nabla(!(\nabla(n\backslash s))/n)), !(\nabla(\nabla n\backslash n)), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ contr_2$$

$$\dfrac{n \longrightarrow n \qquad !(\nabla n), !(\nabla(!(\nabla(n\backslash s))/n)), !(\nabla(\nabla n\backslash n)), !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}{!(\nabla n), !(\nabla(!(\nabla(n\backslash s))/n)), !(\nabla(\nabla n\backslash n))/n, n, !(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s) \longrightarrow s, s}\ /_L$$

Philippa likes her dad, Ada does too.

Strict vs Sloppy

$$\frac{\mathbf{D}}{\nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),n,n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ contr_2$$

$$\frac{\overline{\nabla n\longrightarrow\nabla n}\qquad \nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),n,n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),n,\ \nabla n,\ \nabla n\backslash n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ \backslash_L$$

$$\frac{\overline{\nabla n\longrightarrow\nabla n}\qquad \nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),n,\ \nabla n,\ \nabla n\backslash n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\nabla n\backslash n,\ \nabla n,\ \nabla n\backslash n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ \backslash_L$$

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$$\frac{\nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\nabla n\backslash n,\ \nabla n,\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ \nabla_L$$

$$\frac{\nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\ \nabla(\nabla n\backslash n),\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ perm'$$

$$\frac{\nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla n,\ \nabla(\nabla n\backslash n),\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,\ \nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla(\nabla n\backslash n),\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ perm$$

$$\frac{\nabla n,\ \nabla n,!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla(\nabla n\backslash n),\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{!(\nabla n),!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla(\nabla n\backslash n),\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ contr_2$$

$$\frac{!(\nabla n),!(\nabla(!(\nabla(n\backslash s))/n)),\ \nabla(\nabla n\backslash n),\ \nabla(\nabla n\backslash n),\ \nabla n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{!(\nabla n),!(\nabla(!(\nabla(n\backslash s))/n)),!(\nabla(\nabla n\backslash n)),\ \nabla n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ contr_2$$

$$\frac{!(\nabla n),!(\nabla(!(\nabla(n\backslash s))/n)),!(\nabla(\nabla n\backslash n)),\ \nabla n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{!(\nabla n),!(\nabla(!(\nabla(n\backslash s))/n)),!(\nabla(\nabla n\backslash n)),!(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ contr_2$$

$$\frac{\overline{n\longrightarrow n}\qquad !(\nabla n),!(\nabla(!(\nabla(n\backslash s))/n)),!(\nabla(\nabla n\backslash n)),!(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{!(\nabla n),!(\nabla(!(\nabla(n\backslash s))/n)),!(\nabla(\nabla n\backslash n))/n,n,!(\nabla n),\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ /_L$$

$$\frac{\overline{n\longrightarrow n}\qquad \overline{s,s\longrightarrow s,s}}{s,n,n\backslash s\longrightarrow s,s}\ \backslash_L$$

$$\frac{\overline{n\longrightarrow n}\qquad s,n,n\backslash s\longrightarrow s,s}{n,n\backslash s,n,n\backslash s\longrightarrow s,s}\ \backslash_L$$

$$\frac{n,n\backslash s,n,n\backslash s\longrightarrow s,s}{n,n\backslash s,\ \nabla n,n\backslash s\longrightarrow s,s}\ \nabla_L$$

$$\frac{n,n\backslash s,\ \nabla n,n\backslash s\longrightarrow s,s}{n,\ \nabla(n\backslash s),\ \nabla n,n\backslash s\longrightarrow s,s}\ \nabla_L$$

$$\frac{n,\ \nabla(n\backslash s),\ \nabla n,n\backslash s\longrightarrow s,s}{\nabla n,\ \nabla(n\backslash s),\ \nabla n,n\backslash s\longrightarrow s,s}\ \nabla_L$$

$$\frac{\nabla n,\ \nabla(n\backslash s),\ \nabla n,n\backslash s\longrightarrow s,s}{\nabla n,!(\nabla(n\backslash s)),\ \nabla n,n\backslash s\longrightarrow s,s}\ contr_1$$

$$\frac{\overline{\nabla(n\backslash s)\longrightarrow\nabla(n\backslash s)}\qquad \nabla n,!(\nabla(n\backslash s)),\ \nabla n,n\backslash s\longrightarrow s,s}{\nabla n,!(\nabla(n\backslash s)),\ \nabla n,\ \nabla(n\backslash s),\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ \backslash_L$$

$$\frac{\nabla n,!(\nabla(n\backslash s)),\ \nabla n,\ \nabla(n\backslash s),\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,!(\nabla(n\backslash s)),\ \nabla(n\backslash s),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ perm$$

$$\frac{\nabla n,!(\nabla(n\backslash s)),\ \nabla(n\backslash s),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,!(\nabla(n\backslash s)),!(\nabla(n\backslash s)),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ contr_1$$

$$\frac{\overline{n\longrightarrow n}\qquad \nabla n,!(\nabla(n\backslash s)),!(\nabla(n\backslash s)),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,!(\nabla(n\backslash s))/n,n,!(\nabla(n\backslash s)),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ /_L$$

$$\frac{\nabla n,!(\nabla(n\backslash s))/n,n,!(\nabla(n\backslash s)),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,\ \nabla(!(\nabla(n\backslash s))/n),n,!(\nabla(n\backslash s)),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ \nabla_L$$

$$\frac{\overline{n\longrightarrow n}\qquad \nabla n,\ \nabla(!(\nabla(n\backslash s))/n),n,!(\nabla(n\backslash s)),\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,\ \nabla(!(\nabla(n\backslash s))/n),n,!(\nabla(n\backslash s))/n,n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ /_L$$

$$\frac{\nabla n,\ \nabla(!(\nabla(n\backslash s))/n),n,!(\nabla(n\backslash s))/n,n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,\ \nabla(!(\nabla(n\backslash s))/n),n,\ \nabla(!(\nabla(n\backslash s))/n),n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ \nabla_L$$

$$\mathbf{D} = \frac{\nabla n,\ \nabla(!(\nabla(n\backslash s))/n),n,\ \nabla(!(\nabla(n\backslash s))/n),n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}{\nabla n,\ \nabla(!(\nabla(n\backslash s))/n),\ \nabla(!(\nabla(n\backslash s))/n),n,n,\ \nabla n,\ \nabla(n\backslash s)\backslash(n\backslash s)\longrightarrow s,s}\ perm$$

Philippa likes her dad, Ada does too.

Semantics of Pregroups

$$\mathcal{B} = \{n, s\}$$

Semantics of basic types

$$[\![n]\!] := N, [\![s]\!] := S$$

Semantics of complex types

$$[\![x \cdot y]\!] := [\![x]\!] \times [\![y]\!]$$

$$[\![x^r \cdot y]\!] := [\![y]\!]^{[\![x]\!]} \qquad [\![y \cdot x^l]\!] := [\![y]\!]^{[\![x]\!]}$$

Semantics of Pregroups

The semantics is non-compositional

$$[\![x^r \cdot y]\!] := [\![y]\!]^{[\![x]\!]} \neq [\![x^r]\!] \times [\![y]\!]$$

$$[\![y \cdot x^l]\!] := [\![y]\!]^{[\![x]\!]} \neq [\![y]\!] \times [\![x^l]\!]$$

A consequence is ambiguity

$$[\![(x \cdot y) \cdot z^l]\!] = [\![x \cdot y]\!]^{[\![z]\!]} = ([\![x]\!] \times [\![y]\!])^{[\![z]\!]}$$

$$[\![x \cdot (y \cdot z^l)]\!] = [\![x]\!] \times ([\![y]\!]^{[\![z]\!]})$$

$x \cdot y \cdot z^l = (x \cdot y) \cdot z^l = x \cdot (y \cdot z^l)$

$[\![(x \cdot y) \cdot z^l]\!] \neq [\![x \cdot (y \cdot z^l)]\!]$

Vector Semantics of Pregroups

Semantics of basic types

$$[\![n]\!] := \mathbf{N_k}, \quad [\![s]\!] := \mathbf{S_k}$$

Semantics of complex types

$$[\![x \cdot y]\!] := [\![x]\!] \otimes [\![y]\!]$$

$$[\![x^l]\!] = [\![x^r]\!] := [\![x]\!]^*$$

Recovering function spaces

$$[\![y \cdot x^l]\!] := [\![x]\!]^* \otimes [\![y]\!] \cong Hom([\![x]\!], [\![y]\!])$$
$$[\![x^r \cdot y]\!] := [\![x]\!]^* \otimes [\![y]\!] \cong Hom([\![x]\!], [\![y]\!])$$

$$V^* \otimes W \cong Hom(V, W)$$

Semantics of Pregroups

The semantics is compositional

$$[\![y \cdot x^l]\!] := [\![x]\!]^* \otimes [\![y]\!]$$
$$[\![x^r \cdot y]\!] := [\![x]\!]^* \otimes [\![y]\!]$$

There is no ambiguity

$$[\![(x \cdot y) \cdot z^l]\!] = [\![x \cdot y]\!] \otimes [\![z^l]\!] = Hom([\![z]\!], [\![x]\!] \otimes [\![y]\!]) = [\![z]\!]^* \otimes ([\![x]\!] \otimes [\![y]\!])$$
$$[\![x \cdot (y \cdot z^l)]\!] = [\![x]\!] \otimes [\![y \cdot z^l]\!] = [\![x]\!] \otimes Hom([\![z]\!], [\![y]\!]) = [\![x]\!] \otimes ([\![z]\!]^* \otimes [\![y]\!])$$

$$[\![z]\!]^* \otimes [\![x]\!] \otimes [\![y]\!] \cong [\![x]\!] \otimes [\![z]\!]^* \otimes [\![y]\!]$$

Categorical Semantics

strongly monoidal functor

Pregroup → FdVect

Non Commutative Compact Closed Category

Compact Closed Category

Coecke, Sadrzadeh, Clark, (Lambek's 90th Festschrift), 2010

Preller, Sadrzadeh, JoLLI, 2011

Categorical Semantics

Coecke, Grefenstette, Sadrzadeh, APAL, 2013

Categorical Semantics

Wijnholds Sadrzadeh, CAPNS 2018, JoLLI 2019,
modalities = Frobenius Algebras
Unresolved ambiguities
Impercise interpretations

Categorical Semantics

!L* → Vector Semantics

Kanovich, Kuznetsov, Nigam, Scedrov

$$T_{k_0}V := \bigoplus_{i=0}^{k_0} V^{\otimes i} = \mathbb{R} \oplus V \oplus (V \otimes V) \ \oplus \cdots \oplus V^{\otimes k_0}.$$

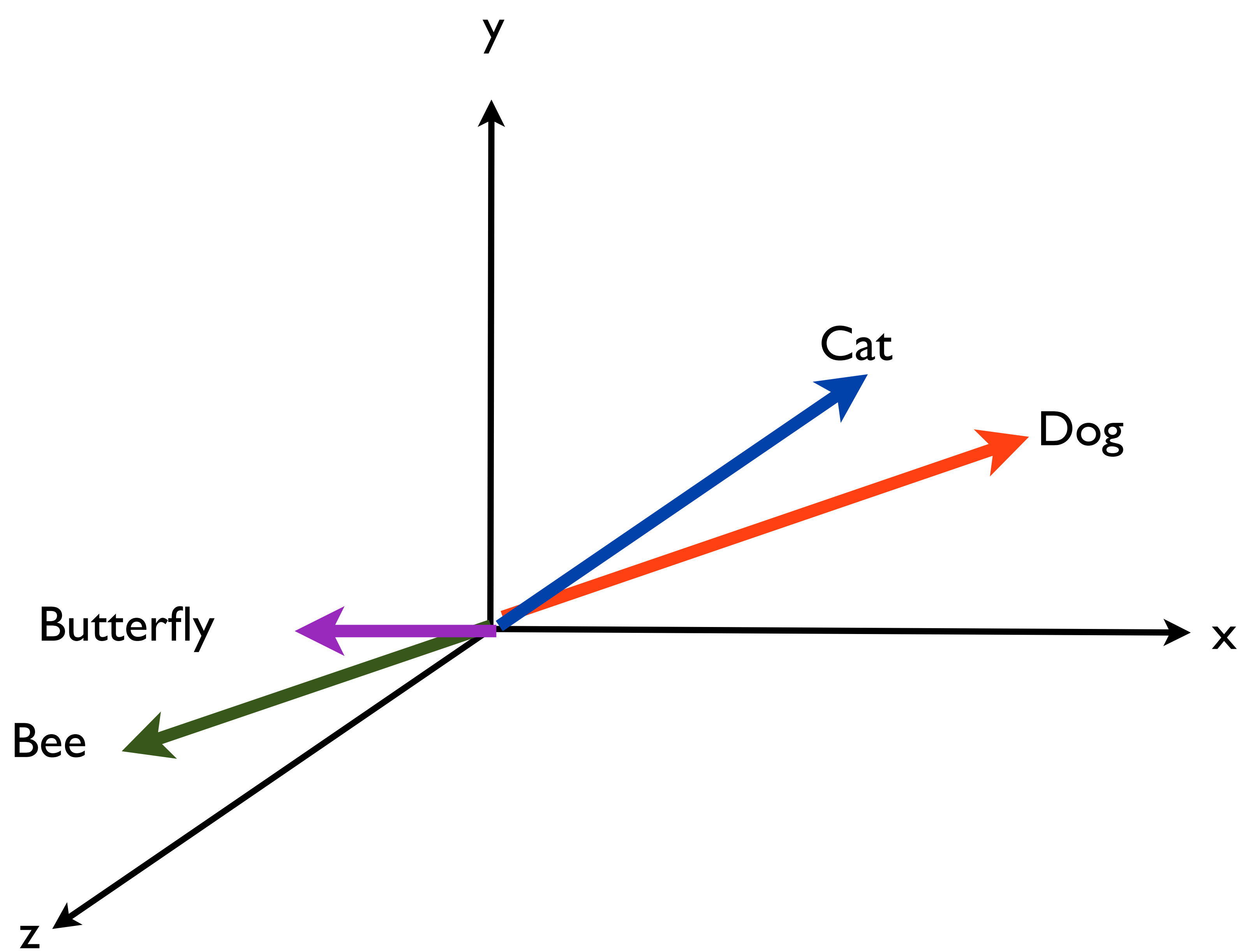
McPheat, Sadrzadeh, Wazni, Toumi, Correia:
ACT 2020
J. Cog Sci (to appear)

McPheat, Sadrzadeh, Wazni:
SemSpace2021
LACL 2021

Natural Language Semantics

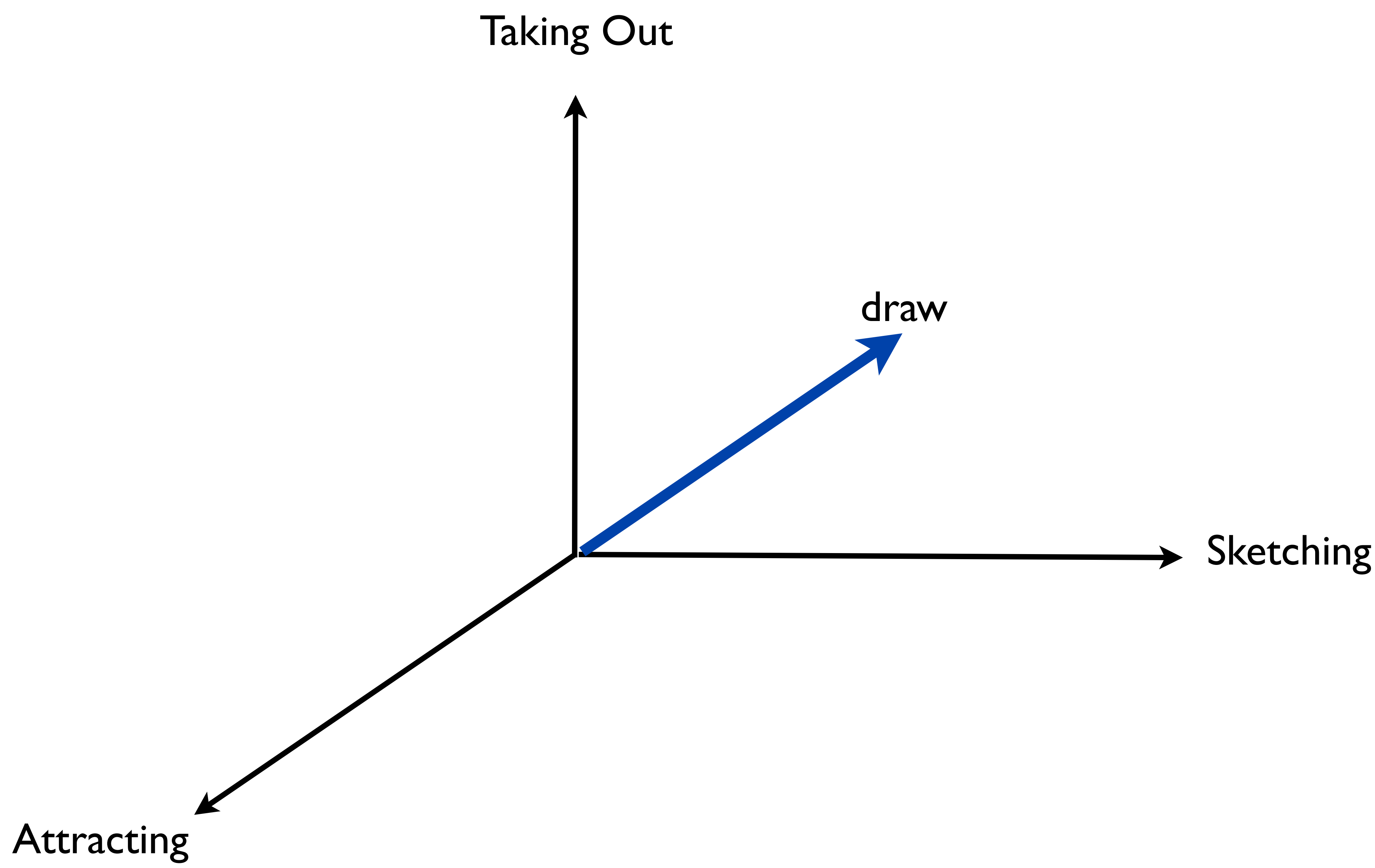
Distributional Semantics of NL: Firth, Harris 1950's

Information Retrieval: Salton, Sparks-Jones, …Document Modelling



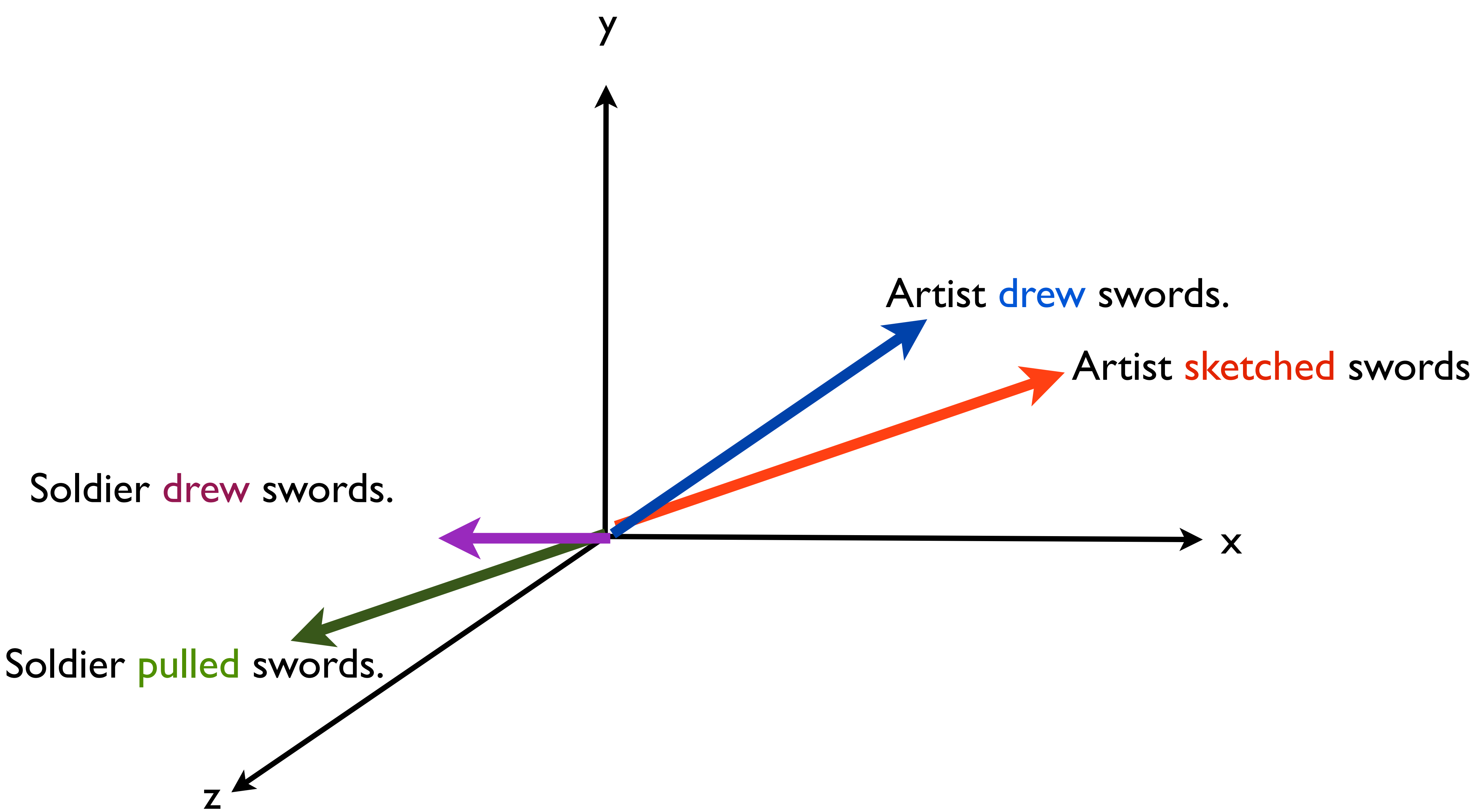
Disambiguation

Lin, Schutze, Lapata, Hill, …1980 till 2010's



Disambiguation

Grefenstette, Sadrzadeh, EMNLP 2011, 2013, JCL 2015



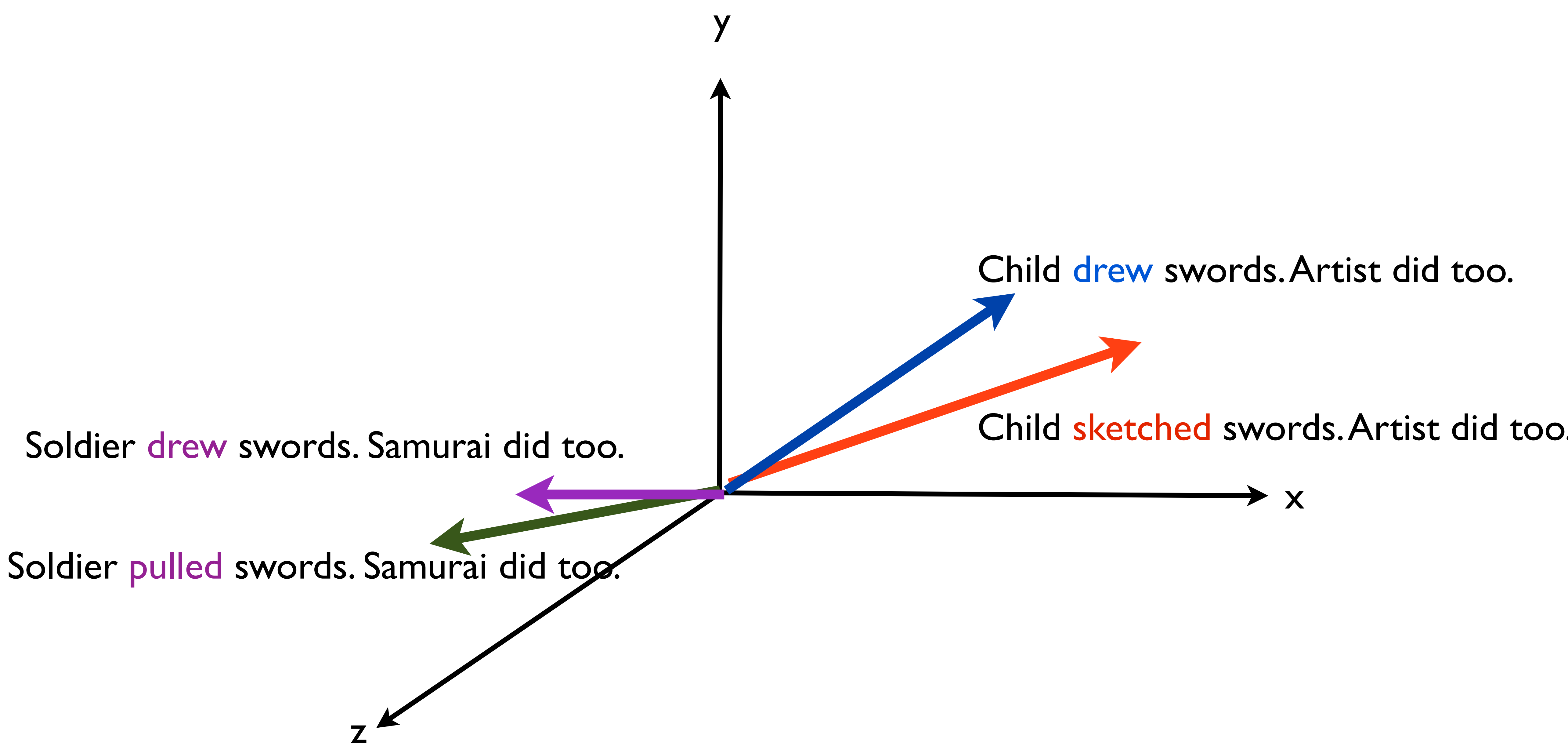
Disambiguation

Mllajevs, Kartsaklis, Sadrzadeh, Purver, ENMLP 2014

Method	GS11	KS14	NWE
Verb only	0.212	0.325	0.107
Addition	0.103	0.275	0.149
Multiplication	0.348	0.041	0.095
Kronecker	0.304	0.176	0.117
Relational	0.285	0.341	0.362
Copy subject	0.089	0.317	0.131
Copy object	0.334	0.331	**0.456**
Frobenius add.	0.261	0.344	0.359
Frobenius mult.	0.233	0.341	0.239
Frobenius outer	0.284	0.350	0.375

Disambiguation

Wijnholds, Sadrzadeh
NAACL 2019



Disambiguation

Wijnholds, Sadrzadeh
NAACL 2019

	CB	W2V	GloVe	FT
Verb Only Vector	.4150	.2260	.4281	.2261
Verb Only Tensor	.3039	.4028	.3636	.3548
Add. Linear	.4081	.2619	.3025	.1292
Mult. Linear	.3205	-.0098	.2047	.2834
Add. Non-Linear	.4125	.3130	.3195	.1350
Mult. Non-Linear	.4759	.1959	.2445	.0249
Best Tensor	**.5078**	.4263	.3556	.4543
2nd Best Tensor	.4949	.4156	.3338	.4278

Disambiguation

Wijnholds, Sadrzadeh, Clark, CoNLL2020

	ML08	**ML10**	**GS11**	**KS13a**	**KS13b**
$\mathbf{v_{skip}}$	0.07	0.40	0.23	0.18	0.45
$\mathbf{V}_{Kron}$	**0.25**	0.40	0.27	**0.26**	0.45
$\mathbf{V}_{Rel}$	0.11	0.43	0.31	0.18	0.47
$\widetilde{\mathbb{V}}^{o/s/s,o}_{(0)}$	0.06	0.53	0.33	0.10	0.64
$\mathbb{V}^{sent\|s/o}_{(1)}$	0.16	-0.00	0.37	0.06	-0.06
$\widetilde{\mathbb{V}}^{o\|s/s\|o}_{(1)}$	0.12	**0.64**	**0.40**	0.22	**0.69**
$\mathbb{V}^{sent\|s,o}_{(2)}$	0.18	0.00	-0.03	0.00	-0.03
SoTA	0.19	0.45	0.46	0.22	0.73
Human	0.66	0.71	0.74	0.58	0.75

Disambiguation

McPheat,, Wazni, Sadrzadeh, LACL 2021

Method	Embeddings	Results
Compositional Copying Tensor	word2vec	0.653
	fasttext	0.610
Verb Only Vector	word2vec	0.583
	fasttext	0.651
Verb Only Tensor	word2vec	0.566
	fasttext	0.533
BERT phrase		0.575

Summary

Pregroup Grammars provide a nice axiomatics that represent intuitive grammatical reductions.

They enjoy a vector space semantics, which relates them to main stream NLP tasks.

The logic of a pregroup is poor, but they are translatable to Lambek Calculus.

Future Directions

The vector semantics has found a translation to Quantum circuits and people used them to run sentence-level experiments in IBMq (Lorentz et al 2020: Coecke's group in CQ).

Extending this to beyond-sentence: RAEng senior fellowship 2022.

The rules for copying and permutation have no connections to the monoid operations.

Vector semantics has always been incomplete.